1 (a) When θ is small, show that $\dfrac{\cos\theta}{\sin\theta}$ can be approximated by $\dfrac{2-\theta^2}{2\theta}$

(b) Hence, approximate the value of $\dfrac{\cos\frac{\pi}{24}}{\sin\frac{\pi}{24}}$

(c) Calculate the percentage error in your approximation

a/ when θ is small $\quad \sin\theta \approx \theta \quad \cos\theta \approx 1 - \frac{\theta^2}{2}$

$$\frac{\cos\theta}{\sin\theta} = \frac{\left(1 - \frac{\theta^2}{2}\right)}{\theta} \times 2 = \frac{2-\theta^2}{2\theta}$$

b/ $$\frac{2-\left(\frac{\pi}{24}\right)^2}{2\left(\frac{\pi}{24}\right)} = 7.573987421 \qquad 7.57 \text{ (3sf)}$$

c/ $$\frac{\cos\left(\frac{\pi}{24}\right)}{\sin\left(\frac{\pi}{24}\right)} = 7.59575\ldots$$

$$\% \text{ error} = \frac{7.59575 - 7.57398\ldots}{7.59575\ldots} \times 100$$

$$= 0.287\% \text{ (3sf)}$$

---

2 (a) When x is small, show that $\tan(3x)\cos(2x)$ can be approximated by $3x - 6x^3$

(b) Hence, approximate the value of $\tan(0.3)\cos(0.2)$

(c) Calculate the percentage error in your approximation

when x is small $\quad \tan x \approx x \quad \cos x \approx 1 - \frac{x^2}{2}$

$$\tan(3x)\cos(2x) \approx (3x)\left(1 - \frac{(2x)^2}{2}\right)$$

$$\approx 3x - 3x\left(\frac{4x^2}{2}\right)$$

$$\approx 3x - 6x^3$$

b/ $3(0.1) - 6(0.1)^3 = \underline{0.294}$

c/ $\tan(0.3)\cos(0.2) = 0.30317\ldots$

$$\% \text{ error} = \frac{0.30317 - 0.294}{0.30317} \times 100$$

$$= \underline{3.02\%}$$

---

3 (a) Given that θ is small, use the small angle approximation of cos θ to show that

$$4\cos(\theta) + \cos^2(2\theta) \approx 5 - 6\theta^2 + 4\theta^4$$

(b) Hence find an approximation of $4\cos(\theta) + \cos^2(2\theta)$ when $\theta = 3°$

(c) Calculate the percentage error in your approximation

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$4\cos\theta + (\cos 2\theta)^2 \approx 4\left(1 - \frac{\theta^2}{2}\right) + \left(1 - \frac{(2\theta)^2}{2}\right)^2$$

$$\approx 4 - 2\theta^2 + (1 - 2\theta^2)^2$$

$$\approx 4 - 2\theta^2 + 1 - 4\theta^2 + 4\theta^4$$

$$\approx \underline{5 - 6\theta^2 + 4\theta^4}$$

b/ $3° = \frac{1}{60}\pi$ radians

$$5 - 6\left(\frac{1}{60}\pi\right)^2 + 4\left(\frac{1}{60}\pi\right)^4 = \underline{4.98358}$$

$$4\cos(3°) + (\cos(6°))^2 = 4.98359$$

$$\% \text{ error} = \frac{4.98359 - 4.98358}{4.98359} \times 100 = \underline{0.000225\%}$$

4 Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{4\theta \sin 3\theta}{1 - \cos 2\theta}$$

$\sin\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{\theta^2}{2}$

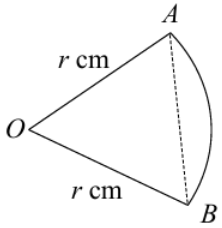
$$\frac{4\theta(3\theta)}{1-\left(1-\frac{(2\theta)^2}{2}\right)}$$

$$\frac{12\theta^2}{1-1+2\theta^2}$$

$$\frac{12\theta^2}{2\theta^2} = 6$$

---

5 Sector AOB is a sector of a circle, radius r cm.



Triangle AOB is equilateral.
The area of sector AOB is 24π cm^2

Find, in terms of π, the perimeter of the sector.

equilateral triangle

$\therefore \theta = \frac{\pi}{3}$

Sector area $= \frac{1}{2} r^2 \theta$

$$24\pi = \frac{1}{2} r^2 \left(\frac{\pi}{3}\right)$$

$$24\pi = \frac{1}{6}\pi r^2$$

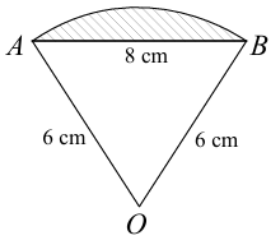
$$r^2 = 144$$

$$r = 12 \text{ cm}$$

perimeter $= r\theta$

$$= \frac{12\pi}{3} = 4\pi \text{ cm}$$

6 Sector AOB is a sector of a circle, radius 6cm. The chord AB is 8cm long.



(a) Find the angle AOB in radians, giving your answer to 3 decimal places
(b) Calculate the area of the sector AOB
(c) Calculate the shaded area.

a) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{6^2 + 6^2 - 8^2}{2(6)(6)}$

$\cos A = \frac{1}{9}$

$A = 1.46$ radians

b) Sector area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(6)^2 (1.46)$

$= 26.3 \text{ cm}^2$

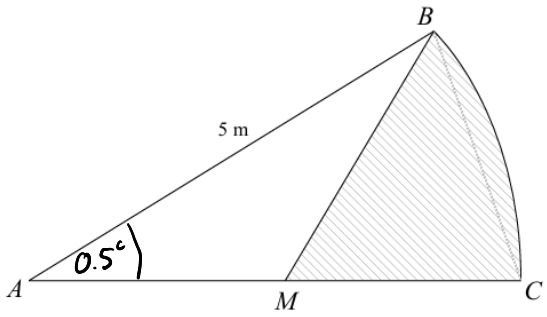
c) Area of triangle $= \frac{1}{2}(6)(6)\sin(1.46)$

$= 17.9 \text{ cm}^2$

Shaded area $= 26.3 - 17.9$

$= 8.38 \text{ cm}^2$

7 Sector ABC is a sector of a circle, centre A and and radius 5m. Angle $BAC = 0.5$ radians



(a) Find the length of the arc BC
(b) Calculate the area of the sector ABC
Given that M is the midpoint of AC
(c) Find the perimeter of the shaded region
(d) Find the area of the shaded region

a) Arc length $= r\theta$

$= 5(0.5)$

$= 2.5$ m

b) Sector area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(5)^2(0.5)$

$= 6.25\ m^2$

c) $a^2 = b^2 + c^2 - 2bc\cos A$

$BM^2 = 5^2 + 2.5^2 - 2(5)(2.5)\cos(0.5)$

$BM^2 = 9.31$

$BM = 3.05$ m

perimeter $= 2.5 + 2.5 + 3.05$

$= 8.05$ m

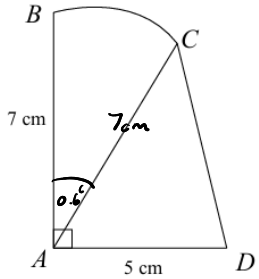
d) Area of triangle ABM $= \frac{1}{2}(5)(2.5)\sin(0.5)$

$= 3.0\ m^2$

Shaded region $= 6.25 - 3.0$

$= 3.25\ m^2$

8 Sector ABC is a sector of a circle, centre A and and radius 7 cm. Angle $BAC = 0.6$ radians



(a) Find the length of the arc BC
(b) Calculate the area of the sector ABC
(c) Find the size of angle CAD, in radians
(d) Find the total area of the shape $ABCD$

a/ Arc length $= r\theta$

$= 7(0.6)$

$= 4.2$ cm

b/ Sector area $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(7)^2(0.6)$

$= 14.7$ cm^2

c/ $CAD = \frac{\pi}{2} - 0.6$

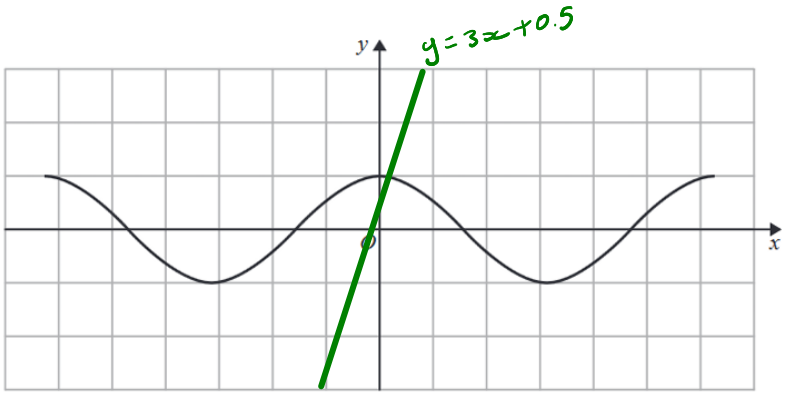
$= 0.971$ radians (3sf)

d/ Area of triangle $CAD = \frac{1}{2}(7)(5)\sin(0.971)$

$= 14.4$ cm^2

$14.4 + 14.7 =$ 29.1 cm^2

9 The diagram shows the graph of $y = \cos x$ where x is measured in radians.



(a) Explain why the equation $\cos x - 3x - 0.5 = 0$ only has one real root.

(b) Given the root of the equation is α, and α is small, use small angle approximations to find the value of α to 3 decimal places.

a/ $y = \cos x$ and $y = 3x + 0.5$ have only one intersection.

b/ when θ is small: $\cos\theta \approx 1 - \frac{\theta^2}{2}$

$$1 - \frac{\alpha^2}{2} - 3\alpha - 0.5 = 0$$

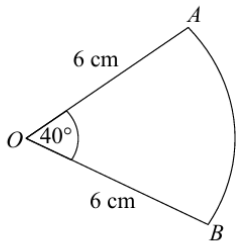
$$0.5 - \frac{\alpha^2}{2} - 3\alpha = 0$$

$$1 - \alpha^2 - 6\alpha = 0$$

$$\alpha^2 + 6\alpha - 1 = 0$$

$$\alpha = 0.162$$

10 Sector AOB is a sector of a circle. A student's attempt to find the area of sector AOB is shown below.



Area $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 6^2 \times 40$

$= 58\ \text{cm}^2$

(a) Identify the mistake made by the student. **(1)**

(b) Write out a correct solution. **(2)**

a/ θ should be in radians, not degrees.

b/ $\theta = \frac{40}{180} \times \pi = \frac{2}{9}\pi$

$\text{Area} = \frac{1}{2}(6)^2\left(\frac{2}{9}\pi\right)$

$= 4\pi\ \text{cm}^2$

---

11 Show that, for small values of x, the graph of $y = 3 + 6\sin\frac{x}{3} - 8\tan\frac{x}{2}$ can be approximated by a straight line.

when x is small $\sin x \simeq x$ $\tan x \simeq x$

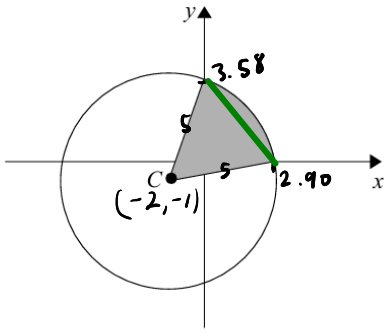
$y = 3 + 6\left(\frac{x}{3}\right) - 8\left(\frac{x}{2}\right)$

$= 3 + 2x - 4x$

$y = 3 - 2x$

straight line as in the form $y = mx + c$

12 The diagram shows a circle, with centre C, with equation $(x + 2)^2 + (y + 1)^2 = 25$
The circle intersects the x-axis at A and intersects the y-axis at B.
The sector ABC is shown on the diagram.



ACB
(a) Shown that angle ~~ABC~~ = 0.958 radians to 3 significant figures.

(b) Find the perimeter of sector ABC.

a/ crosses x when $y = 0$ $\quad (x+2)^2 + (1)^2 = 25$

$(x+2)^2 = 24$

$x = -2 \pm \sqrt{24}$

$= 2.90$

crosses y when $x = 0$ $\quad (2)^2 + (y+1)^2 = 25$

$(y+1)^2 = 21$

$y = -1 \pm \sqrt{21}$

$= 3.58$

length of chord $= \sqrt{2.90^2 + 3.58^2}$

$= 4.61$

$\cos\theta = \dfrac{5^2 + 5^2 - 4.61^2}{2(5)(5)}$

$\cos\theta = 0.5752\ldots$

$\theta = 0.958$ radians

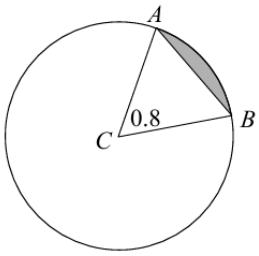
b/ Arc length $= r\theta$

$= 5(0.958)$

$= 4.79$

perimeter $= 5 + 5 + 4.79 = 14.8$ (3sf)

13 The diagram shows a circle, with centre C, with equation $x^2 + y^2 - 6x + 10y = 222$
AB is a chord of the circle.
Angle $ACB = 0.8$ radians.



Find the area of the minor segment shaded on the diagram.

Give your answer to three significant figures.

$(x-3)^2 - 9 + (y+5)^2 - 25 = 222$

$(x-3)^2 + (y+5)^2 = 256$

radius $= \sqrt{256} = 16$

Sector area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(16)^2(0.8)$

$= 102.4$

Triangle area $= \frac{1}{2}(16)(16)\sin(0.8)$

$= 91.8$

shaded segment $= 102.4 - 91.8$

$= 10.6$ units^2

14 Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{5\theta \tan 2\theta}{\cos 4\theta - 1}$$

$\tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{\theta^2}{2}$

$$\frac{5\theta(2\theta)}{1 - \frac{(4\theta)^2}{2} - 1}$$

$$\frac{10\theta^2}{-8\theta^2} = -\frac{5}{4}$$

---

15 (a) For a small angle θ, where θ is in radians, show that $2\cos\theta + (\sin\theta + 1)^2 \approx 2\theta + 3$

(b) Hence determine an approximate solution to $2\cos\theta + (\sin\theta + 1)^2 \approx 20\tan\theta$

$\cos\theta \approx 1 - \frac{\theta^2}{2} \qquad \sin\theta \approx \theta$

a/ $2\left(1 - \frac{\theta^2}{2}\right) + (\theta + 1)^2$

$2 - \theta^2 + \theta^2 + 2\theta + 1$

$2\theta + 3$

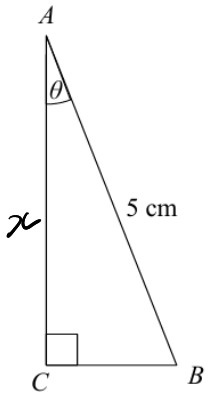
b/ $2\theta + 3 \approx 20\tan\theta$ when θ is small $\tan\theta \approx \theta$

$2\theta + 3 = 20\theta$

$3 = 18\theta$

$\theta = \frac{1}{6}$

16 ABC is a right angled triangle. Angle $BAC = \theta$, where θ is in radians. $AB = 5$ cm.



$\cos\theta \approx 1 - \frac{\theta^2}{2}$

Given that θ is small, find the length of AC.

$\cos\theta = \frac{x}{5}$

$1 - \frac{\theta^2}{2} = \frac{x}{5}$

$x = 5 - \frac{5\theta^2}{2}$

the smaller θ gets AC gets closer to 5.

---

17 The area of a sector of a circle is 1.872 cm^2.
The arc length of the sector is 1.56 cm.
The angle of the sector is θ radians and the radius of the circle is r cm.

Find the value of θ and the value of r.

$\frac{1}{2}r^2\theta = 1.872$ $\quad$ $r\theta = 1.56$

$\theta = \frac{1.56}{r}$

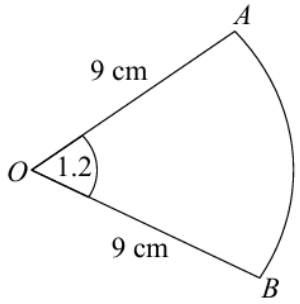
$\frac{1}{2}r^2\left(\frac{1.56}{r}\right) = 1.872$

$0.78r = 1.872$

$r = 2.4$ cm

$\theta = \frac{1.56}{2.4} = 0.65$ radians

18 Sector AOB is a sector of a circle with radius 9 cm. The angle of the sector is 1.2 radians.



(a) Calculate the length of the arc AB

(b) Calculate the area of the sector.

a/ arc length $= r\theta$

$= 9(1.2)$

$= 10.8$ cm

b/ sector area $= \frac{1}{2}(9)^2(1.2)$

$= 48.6\ cm^2$