A Level Maths: Radians

1 (a) When θ is small, show that $\frac{\cos\theta}{\sin\theta}$ can be approximated by $\frac{2-\theta^2}{2\theta}$ **(3)**

(b) Hence, approximate the value of $\frac{\cos\frac{\pi}{24}}{\sin\frac{\pi}{24}}$ **(2)**

(c) Calculate the percentage error in your approximation **(1)**

(Total for question 1 is 6 marks)

---

2 (a) When x is small, show that $\tan(3x)\cos(2x)$ can be approximated by $3x - 6x^3$ **(3)**

(b) Hence, approximate the value of $\tan(0.3)\cos(0.2)$ **(2)**

(c) Calculate the percentage error in your approximation **(1)**

(Total for question 2 is 6 marks)

---

3 (a) Given that θ is small, use the small angle approximation of $\cos\theta$ to show that

$$4\cos(\theta) + \cos^2(2\theta) \approx 5 - 6\theta^2 + 4\theta^4$$

(3)

(b) Hence find an approximation of $4\cos(\theta) + \cos^2(2\theta)$ when $\theta = 3°$ **(2)**

(c) Calculate the percentage error in your approximation **(1)**

(Total for question 3 is 6 marks)

---

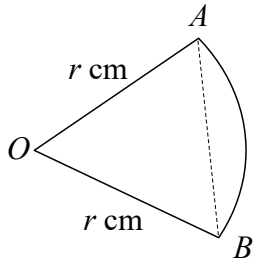
4 Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{4\theta\sin 3\theta}{1-\cos 2\theta}$$

(Total for question 4 is 3 marks)

---

5 Sector AOB is a sector of a circle, radius r cm.



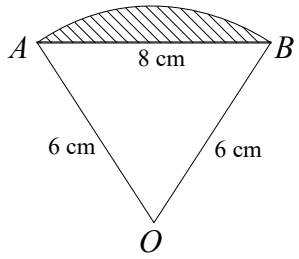
Triangle AOB is equilateral.
The area of sector AOB is 24π cm^2

Find, in terms of π, the perimeter of the sector.

(Total for question 5 is 4 marks)

---

6 Sector AOB is a sector of a circle, radius 6cm. The chord AB is 8cm long.

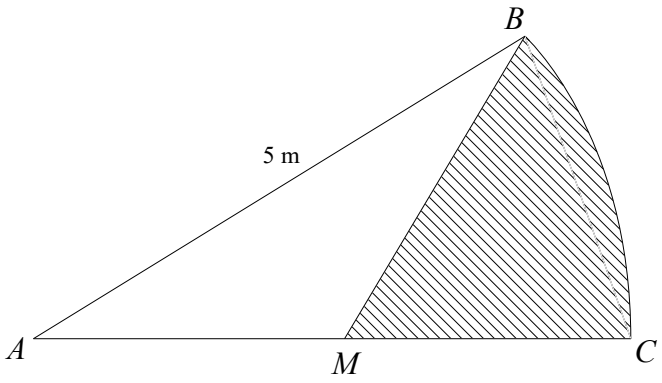


(a) Find the angle AOB in radians, giving your answer to 3 decimal places **(3)**
(b) Calculate the area of the sector AOB **(2)**
(c) Calculate the shaded area. **(3)**

(Total for question 6 is 8 marks)

---

7 Sector *ABC* is a sector of a circle, centre *A* and and radius 5m. Angle *BAC* = 0.5 radians

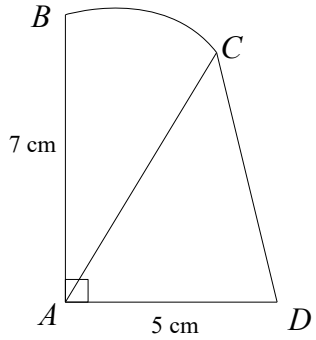


(a) Find the length of the arc *BC* **(2)**
(b) Calculate the area of the sector *ABC* **(2)**
Given that *M* is the midpoint of *AC*
(c) Find the perimeter of the shaded region **(4)**
(d) Find the area of the shaded region **(4)**

(Total for question 7 is 12 marks)

---

8 Sector *ABC* is a sector of a circle, centre *A* and and radius 7 cm. Angle *BAC* = 0.6 radians

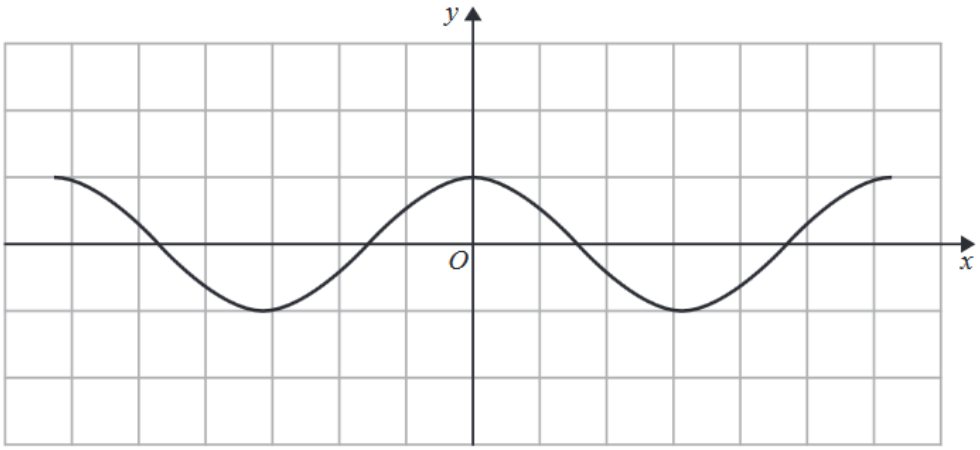


(a) Find the length of the arc *BC* **(2)**
(b) Calculate the area of the sector *ABC* **(2)**
(c) Find the size of angle *CAD*, in radians **(1)**
(d) Find the total area of the shape *ABCD* **(3)**

(Total for question 8 is 8 marks)

---

9 The diagram shows the graph of $y = \cos x$ where x is measured in radians.



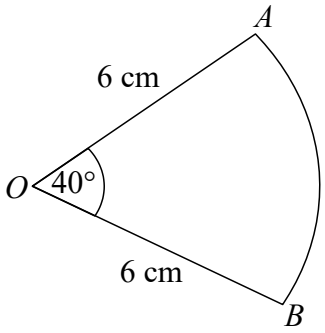
(a) Explain why the equation $\cos x - 3x - 0.5 = 0$ only has one real root. **(2)**

(b) Given the root of the equation is α, and α is small, use small angle approximations to find the value of α to 3 decimal places. **(3)**

(Total for question 9 is 5 marks)

---

10 Sector AOB is a sector of a circle. A student's attempt to find the area of sector AOB is shown below.



$$\text{Area} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2}\times 6^2 \times 40$$
$$= 58 \text{ cm}^2$$

(a) Identify the mistake made by the student. **(1)**

(b) Write out a correct solution. **(2)**

(Total for question 10 is 3 marks)

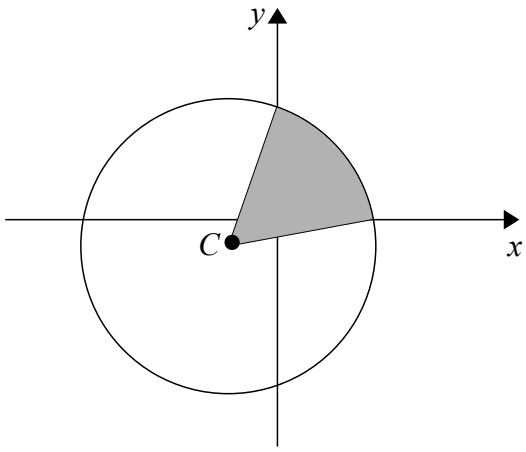
---

11 Show that, for small values of x, the graph of $y = 3 + 6\sin\frac{x}{3} - 8\tan\frac{x}{2}$ can be approximated by a straight line.

(Total for question 11 is 3 marks)

---

12 The diagram shows a circle, with centre C, with equation $(x + 2)^2 + (y + 1)^2 = 25$
The circle intersects the x-axis at A and intersects the y-axis at B.
The sector ABC is shown on the diagram.



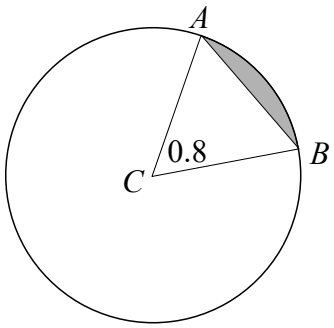
(a) Shown that angle $ACB = 0.958$ radians to 3 significant figures. **(5)**

(b) Find the perimeter of sector ABC. **(3)**

(Total for question 12 is 8 marks)

---

13 The diagram shows a circle, with centre C, with equation $x^2 + y^2 - 6x + 10y = 222$
AB is a chord of the circle.
Angle $ACB = 0.8$ radians.



Find the area of the minor segment shaded on the diagram.

Give your answer to three significant figures.

(Total for question 13 is 5 marks)

---

14 Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{5\theta \tan 2\theta}{\cos 4\theta - 1}$$

(Total for question 14 is 3 marks)

---

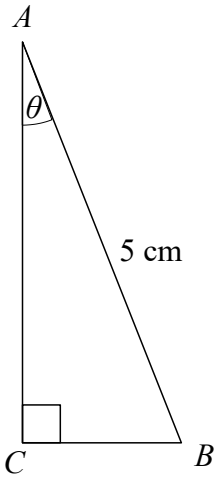
15 (a) For a small angle θ, where θ is in radians, show that $2\cos\theta + (\sin\theta + 1)^2 \approx 2\theta + 3$ **(3)**

(b) Hence determine an approximate solution to $2\cos\theta + (\sin\theta + 1)^2 \approx 20\tan\theta$ **(2)**

(Total for question 15 is 5 marks)

---

16 ABC is a right angled triangle. Angle $BAC = \theta$, where θ is in radians. $AB = 5$ cm.



Given that θ is small, find the length of AC.

(Total for question 16 is 3 marks)

---

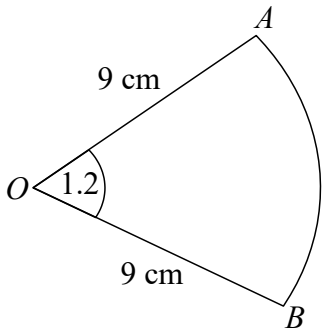
17 The area of a sector of a circle is 1.872 cm^2.
The arc length of the sector is 1.56 cm.
The angle of the sector is θ radians and the radius of the circle is r cm.

Find the value of θ and the value of r.

(Total for question 17 is 3 marks)

---

18 Sector AOB is a sector of a circle with radius 9 cm. The angle of the sector is 1.2 radians.



(a) Calculate the length of the arc AB **(1)**

(b) Calculate the area of the sector. **(2)**

(Total for question 18 is 3 marks)