A Level Maths: Functions and Graphs

1 The functions f and g are defined by

$$f : x \to 3x + 4, \qquad x \in \mathbb{R}$$

$$g : x \to \frac{2}{x+3}, \qquad x \in \mathbb{R},\ x \neq 3$$

(a) Evaluate fg(1) **(2)**

(b) Solve the equation gf(x) = 6 **(4)**

(Total for question 1 is 6 marks)

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2 The function f is defined by

$$f(x) = x^2 + 2x + 1, \quad x \in \mathbb{R}, \quad x \geq -1$$

(a) State the range of f **(1)**

(b) Sketch the graphs of f(x) and $f^{-1}(x)$ on the same diagram **(3)**

(c) Find an expression for $f^{-1}(x)$ and state its domain **(4)**

(Total for question 2 is 8 marks)

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3 The function f is defined by

$$f(x) = 2 + \ln(2x - 1), \quad x \in \mathbb{R}, \quad x > 0.5$$

(a) Find the exact value of ff(1) **(2)**

(b) Find an expression for $f^{-1}(x)$ **(3)**

(Total for question 3 is 5 marks)

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4 The functions f and g are defined by

$$f : x \to e^x, \qquad x \in \mathbb{R}$$

$$g : x \to 2x + \ln x, \qquad x \in \mathbb{R},\ x > 0$$

(a) Write down the range of f **(1)**

(b) Find an expression for the composite function gf **(2)**

(c) Write down the range of gf **(1)**

(Total for question 4 is 4 marks)

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5 The function f is defined by

$$f(x) = \frac{1}{x+2}, \qquad x \in \mathbb{R},\ x \neq -2$$

(a) Write down the range of f(x) **(2)**

(b) Find an expression for $f^{-1}(x)$ and state its domain **(3)**

$$g(x) = x^2 - 5, \qquad x \in \mathbb{R}$$

(c) Solve $fg(x) = \frac{1}{2}$ **(3)**

(Total for question 5 is 8 marks)

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6 The function f is defined by

$$f(x) = x^2 + 4x + 1, \qquad x \in \mathbb{R}$$

(a) Find the range of f(x) **(3)**

(b) Explain why the function f(x) does not have an inverse **(1)**

(Total for question 6 is 4 marks)

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7 (a) Sketch the graph with equation

$$y = |2x - 3|$$

stating the coordinates where the graph meets the coordinate axis. **(2)**

(b) Find the values of x which satisfy $|2x - 3| < 9$ **(2)**

(c) Find the values of x which satisfy $|2x - 3| < x + 1$ **(2)**

(Total for question 7 is 6 marks)

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8 The functions f and g are defined by

$$f : x \to \ln(3x - 2), \qquad x \in \mathbb{R},\ x > \frac{2}{3}$$

$$g : x \to \frac{3}{x-2}, \qquad x \in \mathbb{R},\ x \neq 2$$

(a) Find the exact value of fg(3) **(2)**

(b) Find an expression for $f^{-1}(x)$ and state its domain **(4)**

(c) Sketch the graphs of f(x) and $f^{-1}(x)$ on the same diagram **(3)**

(d) Sketch the graph of $y = |g(x)|$ **(3)**

(e) Find the exact values of $\left|\frac{3}{x-2}\right| = 4$ **(3)**

(Total for question 8 is 15 marks)

9

$$f(x) = \frac{3x + 1}{x - 2} \qquad x \geq 4$$

(a) Find ff(4) **(2)**

(b) State the range of f. **(1)**

(c) Find $f^{-1}(x)$ and state its domain **(3)**

(Total for Question 9 is 6 marks)

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10 (a) (i) Sketch the graph of $y = |x| + 2$

(ii) Explain why $|x| + 2 \geq |x + 2|$ for all real values of x.

(Total for Question 10 is 3 marks)

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11 $f(x) = 2x^2 + 8x + 11 \qquad x \in \mathbb{R}$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found. **(3)**

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point. **(3)**

(c) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where **(2)**

$$g(x) = 2(x - 2)^2 + 8x - 5 \qquad x \in \mathbb{R}$$

(d) Find the range of the function

$$h(x) = \frac{18}{2x^2 + 8x + 11} \qquad x \in \mathbb{R}$$ **(2)**

(Total for Question 11 is 10 marks)

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12 The function f is defined by

$$f(x) = \frac{3x - 5}{x - 2}, \qquad x \in \mathbb{R},\ x \neq 2$$

(a) Find $f^{-1}(4)$ **(2)**

(b) Show that $ff(x) = \frac{ax + b}{x - 1}$ where a and b are integers to be found. **(3)**

(Total for Question 12 is 5 marks)

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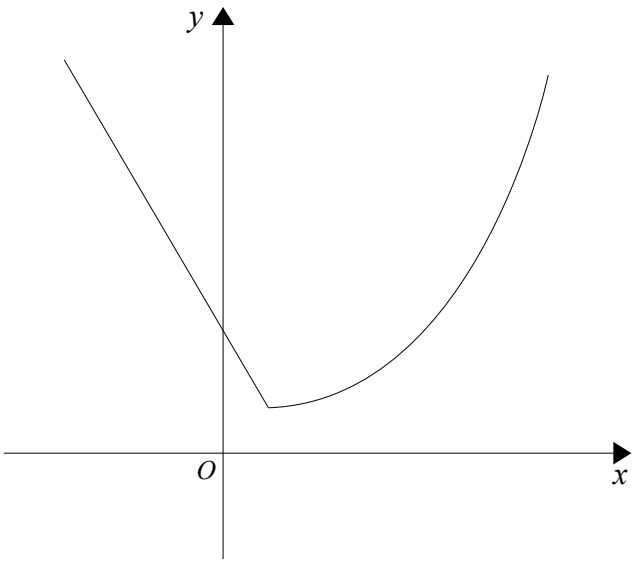
13 The function f is defined by $f(x) = e^{x+1}$, $x \in \mathbb{R}$

Find $f^{-1}(x)$ and state its domain.

(Total for Question 13 is 3 marks)

14



The diagram shows a sketch of $y = g(x)$, where

$$g(x) = \begin{cases} 5 - 3x & x < 1 \\ (x-1)^2 + 2 & x \geq 1 \end{cases}$$

(a) Find the value of gg(0)

(2)

(b) Find all the values of x for which $g(x) > 18$

(4)

The function h is defined as

$$h(x) = (x-1)^2 + 2 \qquad x \geq 1$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$h^{-1}(x) = 2$$

(3)

(Total for Question 14 is 10 marks)

15

$$f(x) = \frac{2x + 1}{x - 1} \qquad x \neq 1$$

(a) Find $f^{-1}(x)$

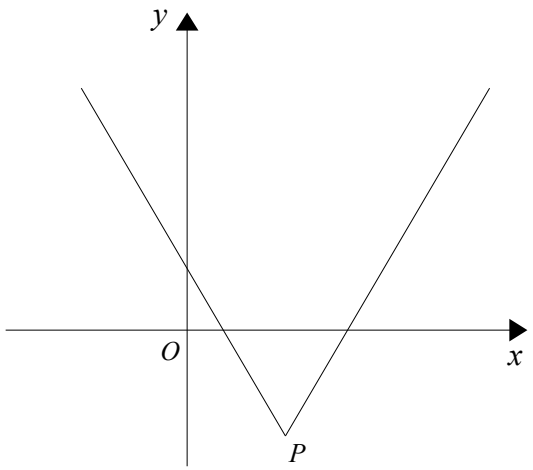
(3)

(b) Find an expression for ff(x)

(1)

(Total for Question 15 is 4 marks)

16



The diagram shows a sketch of $y = 2|x - 3| - 5$

The vertex of the graph is at point P.

(a) Find the coordinates of point P. **(2)**

(b) Solve the equation $10 - x = 2|x - 3| - 5$ **(2)**

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x - 3| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation. **(3)**

(Total for Question 16 is 7 marks)

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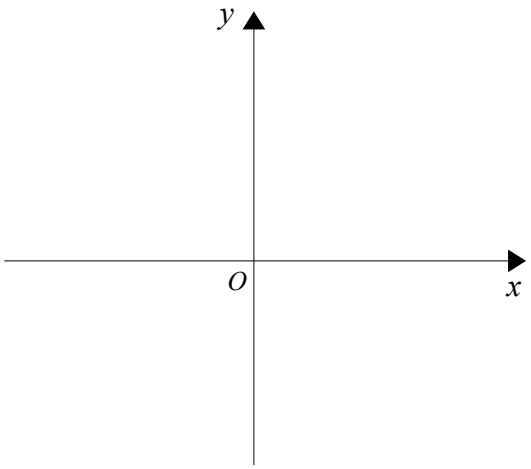
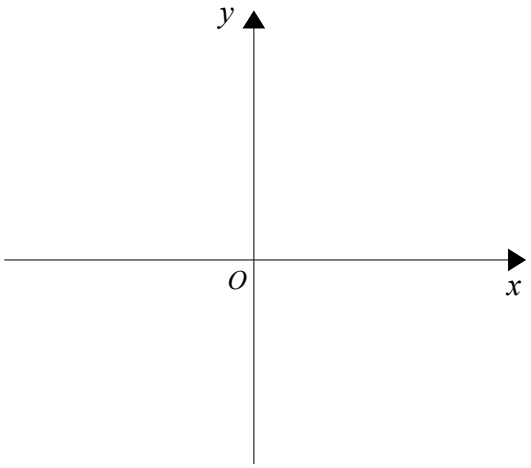
17 Sketch the graph of $y = 1 + |x - 3|$

Show clearly where the graph intersects the axes.



(Total for Question 17 is 2 marks)

18



Sketch the graph of $y = |3x + a|$, where a is a positive constant.

Show clearly where the graph intersects the axes.

(Total for Question 18 is 3 marks)

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19 The function f is defined by $f(x) = 2x^2 + 5, \quad x \geq 0$

(a) Find the range of f. **(1)**

(b) Find $f^{-1}(x)$ **(3)**

(c) State the range of $f^{-1}(x)$ **(1)**

(Total for Question 19 is 5 marks)

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20 $f(x) = \arcsin x$

State the maximum possible domain of f.

(Total for Question 20 is 2 marks)

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21 $f(x) = \dfrac{1}{\sqrt{x + 2}}$

State the maximum possible domain of f.

(Total for Question 21 is 2 marks)

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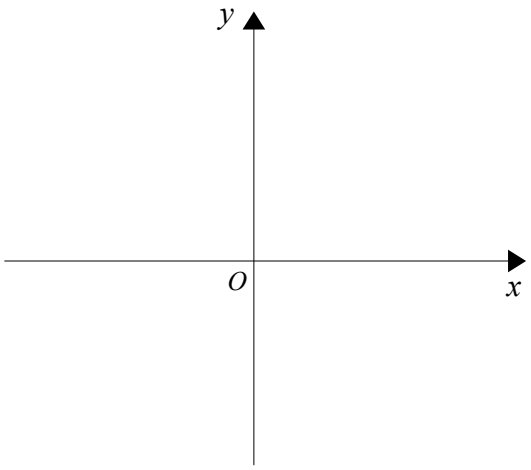
22 (a) Sketch the graph of $y = 2 - |2x + 1|$



(3)

(b) Solve the inequality $2 - |2x + 1| > 0$ **(2)**

(Total for Question 22 is 5 marks)

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23 The function f is defined by $f(x) = 2x^2 - 3x \qquad x \in \mathbb{R}, \ 0 \leq x \leq 4$

(a) Find the range of f. **(3)**

(b) Determine whether f has an inverse.
Fully justify your answer. **(2)**

(Total for Question 23 is 5 marks)

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24 The function f is defined for all real values of x as $f(x) = x^2 - 6x + c$, where c is a constant.

(i) Given that the range of f is $f(x) \geq 2$, find the value of c. **(3)**

(ii) Given instead that $ff(2) = 12$, find the possible values of c. **(4)**

(Total for Question 24 is 7 marks)

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25 Solve the equation $|x + 4| = |2x + 3|$

(Total for Question 25 is 3 marks)

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26 The functions f and g are defined for all real values of x by

$f(x) = x^3$ and $g(x) = x^2 - 1$

(a) Write down expressions for $fg(x)$ and $gf(x)$ **(2)**

(b) Hence find the values of x for which $gf(x) - fg(x) = 18$ **(6)**

(Total for Question 26 is 8 marks)

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27 The function f is defined by $f(x) = (x - 2)^2 - 9$ for $x \geq k$, where k is a constant.

(a) Given that $f^{-1}(x)$ exists, state the least possible value of k. **(1)**

(b) Evaluate ff(5) **(2)**

(c) Solve the equation $f(x) = x$ **(3)**

(d) Explain why your answer to part (c) is also the solution to the equation $f(x) = f^{-1}(x)$ **(1)**

(Total for Question 27 is 7 marks)

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28 (a) Solve the inequality $|x - 1| \leq |2x + 4|$ **(4)**

(b) Give full details of a sequence of two transformations needed to transform the graph of $y = |x - 1|$ onto the graph of $y = |2x + 4|$ **(3)**

(Total for Question 28 is 7 marks)

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29 The functions f and g are defined for all real values of by

$f(x) = 2x^2 - 3x$ and $g(x) = 4x + 6$

(a) Find the range of f. **(3)**

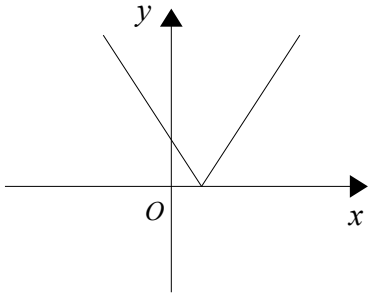
(b) Give a reason why f has no inverse. **(1)**

(c) Given that $gf(2) = g^{-1}(a)$, where a is a constant, determine the value of a. **(3)**

(d) Determine the set of values for which $f(x) > g(x)$. Give your answer in set notation. **(3)**

(Total for Question 29 is 10 marks)

30



The graph shows a sketch of $y = |3x - 2|$

(a) State the coordinates of the points of intersection with the axes. **(2)**

(b) Given that the graphs of $y = |3x - 2|$ and $y = ax + 1$ have two distinct points of intersection, determine

(i) the set of possible values of a, **(4)**

(ii) the x coordinates of the points of intersection of these graphs, giving your answers in terms of a. **(3)**

(Total for Question 30 is 9 marks)

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31 The function $f(x) = \dfrac{e^x}{2 - e^x}$ is defined on the domain $x \in \mathbb{R}, x \neq \ln 2$

(a) Find $f^{-1}(x)$ **(3)**

(b) Write down the range of $f^{-1}(x)$ **(1)**

(Total for Question 31 is 4 marks)

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32 You are given that

$f(x) = x^2 - 4x - 5$ for $x > 2$ and
$g(x) = 2x + 1$

(a) Find $gf(x)$ and state its domain **(3)**

(b) State the range of $gf(x)$. **(1)**

(c) Find $gf^{-1}(x)$. **(5)**

(Total for Question 32 is 9 marks)

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33 The function f is defined for all real values of by

$f(x) = 2x - 5$

(a) Find an expression for $f^{-1}(x)$ **(2)**

(b) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram. **(2)**

(c) Find the set of values for which $f(x) > f^{-1}(x)$. **(2)**

(Total for Question 33 is 6 marks)

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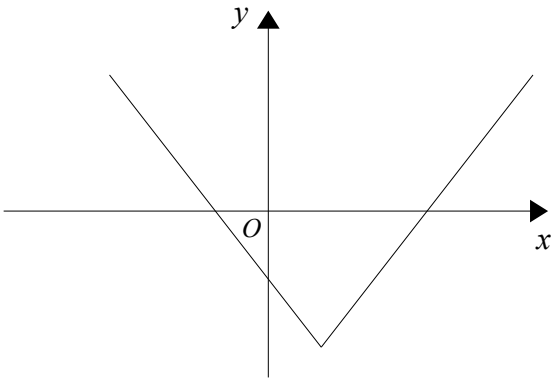
34 The graph shows a sketch of $y = |x - 2| - 3$



Find the set of values for which $|x - 2| < 3$

(Total for Question 34 is 4 marks)

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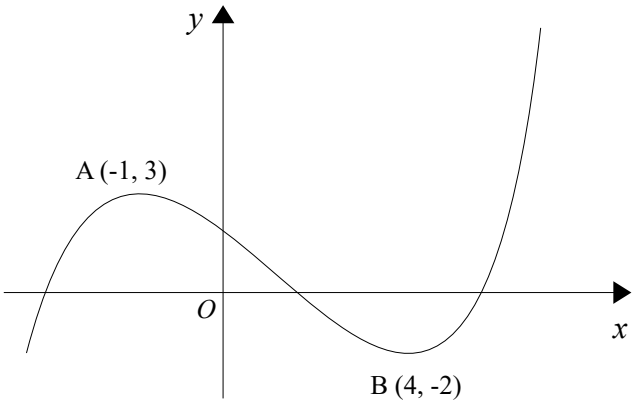
35 The diagram shows the graph of $y = f(x)$



The curve has a maximum point A (-1, 3) and a minimum point B (4, -2).

Showing the coordinates of any stationary points, on separate diagrams sketch the graphs of

(a) $y = 3f(2x)$ **(3)**

(b) $y = f(|x|)$ **(3)**

(c) Write down the constants a and b such that the curve with equation $y = f(x + a) + b$ has a minimum point at the origin O. **(2)**

(Total for Question 35 is 8 marks)

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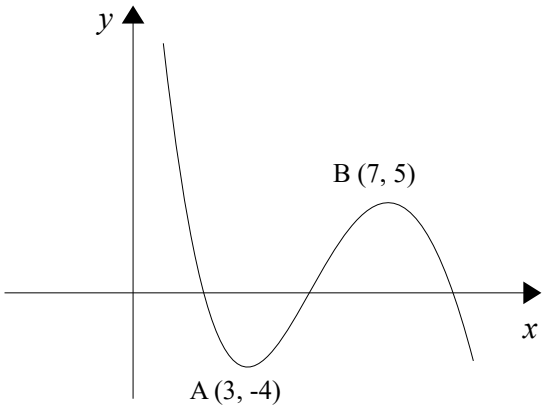
36 The diagram shows the graph of $y = f(x)$



The points A (3, -4) and B (7, 5) are the turning points of the graph.

On separate diagrams sketch the graphs of

(a) $y = 2f(x) - 1$ **(3)**

(b) $y = |f(x)|$ **(3)**

(Total for Question 36 is 6 marks)